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प्रमाण पत्र

प्रमाणित किया जाता है कि प्रो./डॉ./श्री/श्रीमती/सुश्री ......आरती देवी प्र० अ० हिंदी रा० उ० पा० स्कूल ......ने हिमाचल प्रदेश का बृहत् इतिहास विषय पर आयोजित कार्यशाला में संदर्भ प्रवक्ता/विषय विशेषज्ञ/वक्ता के रूप में योगदान दिया। कार्यशाला में ......हिमाचल प्रदेश की सांस्कृतिक विरासत : जिला ऊना के सन्दर्भ में ......शीर्षक पर अपना व्याख्यान दिया/सहभागिता की।

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RESEARCH ARTICLE

Application of Mathematics in Economic Theory

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Abstract:
Application of Mathematics into Economic Theory is a formal description of certain relationships of economic variables; some of these relationships are derived from empirical observations while other ones are deduced from theoretical axioms based on a Rational Economic Agent. Mathematization of Economic Theory broadly started in the second quarter of the 20th Century due to shift of emphasis from partial to general equilibrium analysis, economic dynamism, growth theories and technical progress. In present scenario, it is being widely accepted as a tool in improving Economic Theory and corrective measures for the formulation of Economic Policy; Fiscal and Monetary Policies.

Economic reasoning based on mathematics, in the form of a set of equations and simplified mathematical relationships has been a fundamental factor in the development of Economic Theory as a Science approximating to reality sufficiently closed. Having a fair idea of economic problems with expert knowledge supplemented by appropriate mathematical techniques do provide a better formal insight into the problem and promotes understanding in a systematic and consistent form. Empirical support and mathematical logic makes the theory easily understandable and to form meaningful and testable propositions. That is why Mathematical Methods are being recognized as a rational approach in solving Economic Problems; rational decision making, understanding cause and effect relationship between economic variables, utility and profit maximization, optimum allocation of resources, least cost-combination subject to budget constraint, factor price determination, distribution aspect of the Gross National Product (GNP) and growth path of the economy.

Mathematical Equations in Economic Theory:
While applying Mathematics into Economic Theory, equations can be distinguished in to three parts;
1. Definitional Equations e.g. Profit Maximization behaviour of a Firm or a Producer
Profit (π) = Total Revenue (**TR**) – Total Cost (**TC**)
2. Behavioural Equations e.g. Consumption Behaviour in an Economy $C = 40+0.75\ Y$
3. Equilibrium Equations e.g. determination of Market Equilibrium with Demand and Supply Functions
Demand Function $Q_d = a - b\ P$
Supply Functions $Q_s = c + g\ P$

Differential Equations provide solution to Demand and Supply functions resulting in Equilibrium Price. First Order Difference Equations provide solution to Dynamics of the Equilibrium i.e. how fluctuations converge or diverge to Equilibrium and paves the way for understanding a Business Cycle over a period of time.

Prof. Cournot, mathematically developed equilibrium market condition in view of competition between two sellers referred as **Cournot Duoploy.** Differentiating the profit function with respect to quantity supplied for each firm, **Cournot** deduced a system of linear equations, the simultaneous solution of which gave the equilibrium price, quantity and profit under Duopoly.

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Application of Mathematics in Optimization Theory:
In the simplest form, an optimization problem involves maximizing or minimizing a function, satisfying general necessary and sufficient conditions for optimality. As the main objective of a Consumer is to maximize his Utility from the goods subject to his purchasing power i.e. Budget Constraint. Thus, Utility Optimization is attained by using **Lagrange Multiplier and Bordered Hessian Determinant**

To maximize the Utility Function $U(x_1, x_2)$ subject to the budget constraint $y = p_1x_1 + p_2x_2$
$z = u(x_1, x_2) - \lambda(p_1x_1 + p_2x_2 - y)$
$\partial Z/\partial X_1 = u_1 - \lambda . p_1 = 0$
$\partial Z/\partial X_2 = u_2 - \lambda . p_2 = 0$, Now it follows that $\lambda = \frac{u1}{p1} = \frac{u2}{p2}$
Thus Marginal Utilities should be proportional to prices for consumer equilibrium and hence for maximum utility.
With the help of Partial Derivates, **Slutsky** divided Price Effect into Income effect and Substitution effect of various goods. Generalized form of Slutsky Equation is:-
$\partial q_i/\partial p_i = (\partial q_i / \partial p_i)_{U\text{-constant}} - q_i (\partial q_i/\partial y)_{p\text{-constant}}$
Commodity q_i is defined as Normal good if $\partial q_i/\partial p_i < 0$, Giffen good if $\partial q_i/\partial p_i > 0$ and Inferior good if $\partial q_i/\partial y < 0$
Given the Demand Function: $Q = a - bP$, where P is Price and Q is quantity demanded, then Consumer's Surplus is expressed as $\int_0^q f(q)dq - P.Q$
Given the Supply Function: $Q = c + gP$, where P is Price and Q is quantity demanded , then Producer's Surplus is expressed as $P.Q - \int_0^q f(q)dq$
As the main objective of of an Entrepreneur is to maximize his Profits.
Profit (π) = Total Revenue (TR) – Total Cost (TC)
In order to Maximize Profits, First order condition is: $d\pi/dQ = 0$
and Second order condition is: $d^2\pi / dQ^2 < 0$
For Profit Maximization subject to Budget Constraint, **Lagrange Multiplier** ($\partial Z/\partial X_n = 0$) and **Bordered Hessian determinant** (($\partial^2 Z$ **is negative,** H_1 is +ve) is applied.
Other objective of an Entrepreneur is to minimize his cost of production.
In order to Minimize Cost, First order condition is $d\pi/dQ = 0$
and Second order condition is $d^2\pi / dQ^2 > 0$
For Cost Minimization subject to Budget Constraint, **Lagrange Multiplier** ($\partial Z/\partial X_n = 0$) and **Bordered Hessian determinant** ($\partial^2 Z$ **is positive,** H_1 **is -ve**) is applied.

Application of Mathematics in Production and Distribution:
Production itself can be described as a conversion of Inputs into output.
C.W. Cobb and P.H. Douglas introduced a general production Function: $Q = A\ K^{\alpha} L^{\beta}$
Q is total output, K is the Capital input and L is the Labour input. A is efficiency parameter, α and β are output elasticities. The main feature of this function is that it is homogeneous of degree one ($\alpha + \beta = 1$), depicting constant returns to scale of Production.

Kenneth Arrow, Hollis Chenery, Bagicha Minhas and Robert Solow introduced another Production Function which is linearly homogeneous and having the constant elasticity of Substitution named as **C.E.S. Production Function.**
$Q = A[\alpha K^{-\beta} + (1-\alpha) L^{-\beta}]^{-1/\beta}$

This function being linearly homogeneous, depicts constant returns to scale and qualifies for the application of **Adding-up Theorem/ Euler's Theorem**, describing that if each factor is paid according to its marginal product, then the total product will just get exhaust, thereby providing a solution to the distribution of National Product in the Economy.

In Production, Least-Cost combination is attained when ratio of Marginal Productivity of factors of production equates to ratio of factor prices: $MP_L / MP_K = w / r$

Application of Mathematics in Growth Theory and Fluctuations:
Capital Formation is a necessary condition for Economic Growth and Development. Capital Formation is a the process of adding to a given stock of capital (dk / dt) or the rate of net investment flow $I(t)$ over the period of time t.

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Therefore, $dk/dt = I(t)$ or $k(t) = \int \frac{dk}{dt}.dt = \int I(t)dt$
Income from Investment $Y = \sum_{t=0}^{n} a^t / (1+r)^t$
Present Value (a^t) of any Investment depends upon the rate of interest (**r**) and its frequency of being added up in **n** years.

Growth Path of **Herrod-Domar and Solow Model** describes that capital grows at a rate of $g = s / v$ where **s** is the propensity to save and **v** is the capital –output ratio).
Further, Differential Equations Model of **Dynamic Multiplier** deduces that Investment has a multiplier effect on the level of Income, output and employment in an economy.

Herrod and Domar propounded their Multiplier-Accelerator Theory describing that for full employment equilibrium, rate of growth of Labour force termed as natural rate of growth should be equal to the relative value of saving and capital-output ratio ($n = s / v$)

By using Second Order Difference Equations, **Paul Samuelson** developed his Multiplier-Accelerator Interaction Theory describing that different combination of s **(propensity to save)** and β **(accelerator)** will generate different time-paths of national Income leading to convergent/explosive non-cyclic/explosive cyclic /damped or regular growth path over the period of time.

The Cobweb Theory on the basis of Static Demand Function ($X_{Dt} = a - \alpha P_t$) and Lagged Supply Function ($X_{St} = - b + \beta P_{t-1}$), by using Difference equations, described that growth path over time tend to be oscillatory in terms of explosive oscillations, damped oscillations or regular oscillations.

Application of Mathematics in Input-Output Analysis:
Russian born Economist **Wassily W. Leontief** developed Input-Output Analysis by using Technology Coefficient Matrix thereby providing a solution to the problem that what level of output of each producing sector in an economy can bring about equilibrium for its product in the economy as a whole. **Hawkins and Simon**, further deduced viability of the General Solution for Input-Output Analysis.

Application of Mathematics in Game Theory:
Game theory propounded by **John Von Newmann** introduces the possibility of more than one decision –maker, wherein optimum value of the Objective function for any one decision maker depends not only on his choice but also on the choice of the others. Some players may be risk averters while the others may be risk lovers. Value of the game is just the outcome when both/all the players follow their best strategy thereby reaching to a **Saddle Point Solution** in a Pure Strategy. However, in a mixed strategy, solution of the game is attained by Dominance Probability Method, or Linear Programming Method. Almost every Economic Game Theory serves to support an economic policy.

Application of Mathematics in Linear Programming:
Linear Programming is applied to determine optimal allocation to meet out the given objectives may be to maximize or minimize it subject to a number of constraints in the form of linear inequalities.

The original problem is referred as a Primal Problem. If the Primal Problem requires maximization, the Dual problem is one of minimization. However, value of Dual Objective function will always be greater than or equal to the value of the Primal Objective function termed as **Kuhn Tucker Theorem.** To find out optimal solution of the objective function, we move from one basic feasible solution to another basic feasible solution until the Optimal Solution is achieved. Extensions to Non-linear Optimization with inequality constraints were developed by **Albert W. Tucker** and **Harold Kuhn** in 1951.

It may be concluded that in order to explain the rules of economic theory, mathematical methods based on economic reasoning have been widely used in providing solution to a number of economic problems as it is reflected from the work of many economists, thereby this way it proves that, in practice, mathematics is not a substitute but a complement for the economic sciences, offering new sources for theories, postulates, axioms, theorizations, and explanations. As test of a Theory lies in its empirical verification, therefore, new Economic Theories, new concepts and axioms in Economics, passed through the filter of mathematical logic with empirical support are easily

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understandable and to form meaningful and testable propositions thereby providing a high level of credibility and its impact in the Economy. Economics and Mathematics obtaining this way, has propelled both Mathematics and Economics amongst the noble sciences.

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